Transformations of Functions

**GOAL** Identify the effects of transformations on the graphs of quadratic, exponential, absolute value, and radical functions.

Sometimes a basic function can be related to other functions through transformations. In such instances, the basic function is called a **parent function**.

In general, a related graph is called a **translation** of a parent graph \( y = f(x) \) if the related graph can be given by an equation of the form \( y = k + f(x - h) \). For example, the parent graph \( y = |x| \) is shown below translated vertically and horizontally. Notice that subtracting a positive number from one of the variables moves the graph that number of units in a positive direction parallel to the axis of the variable.

![Graph of parent function |x| translated](image)

**EXAMPLE 1** Graphing a Function Using a Parent Function

Graph the function by translating the graph of its parent function.

a. \( y = 2^x - 3 \)

b. \( y - 2 = (x + 1)^2 \)

**SOLUTION**

To find the parent function, think of the given function with any added or subtracted constants removed.

a. The parent function is \( y = 2^x \).
   Translate the graph of the parent function 3 units to the right.

b. The parent function is \( y = x^2 \).
   Translate the graph of the parent function 2 units up and 1 unit to the left.
In Example 1 you identified the parent function in order to draw the graph of a related function. You can also do the reverse. Given the graph of a related function, you can write the function represented by the graph by first identifying the parent function.

**Example 2 Writing a Function Using a Parent Function**

Tell whether the function represented by the graph has a parent function of \( y = \sqrt{|x|} \), \( y = \sqrt{x} \), \( y = x^2 \), or \( y = 2^x \). Write the function represented by the graph, using the graph of the parent function.

**Solution**

First identify the parent function. Because the shape of the graph is that of a square-root graph, the parent function is \( y = \sqrt{x} \). The graph is translated 2 units to the right, so the function is \( y = \sqrt{x - 2} \).

**Dilations**

You can use the graph of a parent function \( y = f(x) \) to sketch the graph of the related function \( y = a \cdot f(x) \), where \( a \) is nonzero.

- If \( |a| > 1 \), the graph is stretched vertically.
- If \( |a| < 1 \), the graph is shrunk vertically.

The transformations described above are called *dilations*. If \( a = -1 \), the graph of \( y = a \cdot f(x) \) is the mirror image of the graph of \( y = f(x) \) across the \( x \)-axis. This type of transformation is called a *reflection* in the \( x \)-axis. If \( a < 0 \) and \( a \neq -1 \), you have a combination of a dilation and a reflection. The following graph shows the parent graph \( y = x^2 \), the dilations \( y = 2x^2 \) and \( y = \frac{1}{2}x^2 \), and the reflection \( y = -x^2 \).
**EXAMPLE 3**  
**Sketching the Graph of a Dilated Function**

Sketch the graph of the function, using the graph of its parent function.

a. \( y = 4 \cdot 3^x \)

**Solution**

a. First graph the parent function, \( y = 3^x \). Then identify several key points, such as \((0, 1)\) and \((1, 3)\). Since \(|4| > 1\), increase the \(y\)-coordinate of each key point by a factor of 4. Plot and connect the new points.

b. First graph the parent function, \( y = \frac{1}{3} |x| \). Identify several key points on the graph of the parent function, such as \((-3, 3)\), \((0, 0)\), and \((3, 3)\). Since \(|-\frac{1}{3}| < 1\) and \(-\frac{1}{3} < 0\), decrease the \(y\)-coordinate of each key point by a factor of \(\frac{1}{3}\) and reflect the points in the \(x\)-axis. Plot and connect the new points.

**EXERCISES**

Graph the function.

1. \( y = (x + 3)^2 \)
2. \( y - 4 = \sqrt{x + 2} \)
3. \( y + 1 = |x - 4| \)
4. \( y - 3 = \sqrt{x - 1} \)
5. \( y = 3|x| \)
6. \( y = -4^x \)
7. \( y = \frac{1}{4}(x - 1)^2 \)
8. \( y = -\frac{1}{2} |x| \)
9. \( y = 2 \cdot 3^x + 1 \)

Tell whether the graph has a parent function of \( y = |x| \), \( y = \sqrt{x} \), \( y = x^2 \), or \( y = 2^x \). Then write the function represented by the graph.

10.

11.

12.

13.

14.

15.
The graph of a function $y = f(x)$ is given. Graph the related function given. (Hint: First draw the translation of several key points on the graph.)

16. Related function: $y - 2 = f(x)$
17. Related function: $y = f(x + 3)$
18. Related function: $y + 4 = f(x - 1)$

In Exercises 19–21, use the following information.
The running velocity $v$ (in feet per second) that a pole vaulter must reach at launch in order to vault a height $h$ (in feet) can be modeled by $v = 8\sqrt{h}$.

19. What is the parent function of the given function?
20. Graph the given function.
21. Use your graph to estimate the height a vaulter could vault if the vaulter’s running velocity at launch is 30 feet per second.

In Exercises 22–24, use the following information.
The total population $b$ of a certain bacteria can be modeled by $b = 500 \cdot 2^t$, where $t$ is the number of times the bacteria population doubles.

22. What is the parent function of the given function?
23. Graph the given function.
24. Use your graph to estimate the number of times the bacteria population needs to double to reach a total population of 10,000.

In Exercises 25–27, use the following information.
A stone is dropped from a bridge that is 60 feet above the water below. The height $h$ (in feet) of the stone above the water after $t$ seconds can be modeled by $h = -16t^2 + 60$.

25. What is the parent function of the given function?
26. Graph the given function.
27. Use your graph to estimate the number of seconds it takes for the stone to hit the water.

28. Consider the graph of a function $y = a \cdot f(x)$, where $a$ is nonzero. What is the relationship between points on the $x$-axis in the graph of $y = a \cdot f(x)$ and points on the $x$-axis in the graph of the parent function $y = f(x)$?

29. Explain how you could rewrite the equation $y = 5\sqrt{4x - 8} + 1$ so that it is in the form $y - k = a\sqrt{x - h}$. 

988 Appendix 1
Modeling Data with Functions

**GOAL**  
Model data using linear, quadratic, or exponential functions; estimate the correlation coefficient for a set of data.

Data can sometimes be modeled by a function. Drawing a scatter plot of the data can help you recognize the type of function that best models the data. You can then use one of the regression features on a graphing calculator to find and graph an equation of the best-fitting model.

**EXAMPLE 1  Choosing a Model for Data**

Use a graphing calculator to draw a scatter plot of the data. Then tell whether a linear, quadratic, or exponential function would best model the data.

**a.** Average total cost \( y \) of a year of college, where \( x = 3 \) represents 1993:

<table>
<thead>
<tr>
<th>( x )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>7931</td>
<td>8306</td>
<td>8800</td>
<td>9206</td>
<td>9588</td>
<td>10,076</td>
<td>10,444</td>
<td>10,876</td>
</tr>
</tbody>
</table>

**b.** Number of customers \( y \) in a restaurant each hour, where \( x = 3 \) represents 3 P.M.:

<table>
<thead>
<tr>
<th>( x )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>15</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>45</td>
<td>42</td>
<td>31</td>
<td>18</td>
</tr>
</tbody>
</table>

**c.** Population \( y \) of bacteria in a petri dish after \( x \) hours:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>15</td>
<td>35</td>
<td>80</td>
<td>300</td>
<td>740</td>
</tr>
</tbody>
</table>

**SOLUTION**

**a.** The points lie nearly in a straight line. This suggests a linear model.

**b.** The points show a parabolic trend. This suggests a quadratic model.

**c.** The points lie in a curve that seems to have an asymptote. This suggests an exponential model.

---

**Study Tip**  
To be sure that an exponential model fits a set of points \((x, y)\), graph the points \((x, \ln y)\). The new points should fit a linear pattern.
EXAMPLE 2  Finding a Model for Data

Find and graph an equation of the best-fitting model for each data set in Example 1.

SOLUTION

Perform the chosen type of regression on a graphing calculator. Round the values to three significant digits. Then graph the model with the data.

a. The linear regression equation is \( y = 423x + 6660 \).

b. The quadratic regression equation is \( y = -2.55x^2 + 33.5x - 62.7 \).

c. The exponential regression equation is \( y = 1.31 \cdot 2.91^x \).

EXAMPLE 3  Using a Model to Make Predictions

Use the models in parts (a) and (b) of Example 2.

a. Predict the average total cost of a year of college in 2007.

b. Predict the number of customers in the restaurant at 6:30 P.M.

SOLUTION

a. Substituting 17 for \( x \) in the model \( y = 423x + 6660 \) gives \( y = 13,851 \). You can predict that in 2007 the average total cost of a year of college will be about $13,900.

b. Substituting 6.5 for \( x \) in the model \( y = -2.55x^2 + 33.5x - 62.7 \) gives \( y = 47.3125 \). You can estimate that at 6:30 P.M., there were 47 customers in the restaurant.
The **correlation coefficient** \( r \) for a set of paired data is a measure of how well a linear function models the data. If all of the graphed data pairs lie exactly on a line with a positive slope, the correlation coefficient is 1. If all of the graphed data pairs lie exactly on a line with a negative slope, the correlation coefficient is \(-1\). If the graphed data pairs tend not to lie on any line, the correlation coefficient is close to 0.

### Example 4: Estimating a Correlation Coefficient

Estimate the correlation coefficient for the data.

**Solution**

**a.** The scatter plot shows a weak negative correlation, so \( r \) is between 0 and \(-1\), but not too close to either one. An estimate is \( r \approx -0.5 \).

(Actual value: \( r \approx -0.56153 \))

**b.** The scatter plot does not seem to show a correlation, so an estimate is \( r \approx 0 \).

(Actual value: \( r \approx 0.01926 \))

**c.** The scatter plot shows a strong positive correlation. An estimate is \( r \approx 0.9 \).

(Actual value: \( r \approx 0.96019 \))

### Exercises

Tell whether a linear, quadratic, or exponential function would best model the data.

1. 
2. 
3. 
4. 
5. 
6.
Use a graphing calculator to find an equation of the best-fitting model for each data set. Then graph the model with the data.

7. 

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1.6</td>
<td>3</td>
<td>6</td>
<td>13</td>
<td>26</td>
<td>51</td>
<td>102</td>
<td>205</td>
</tr>
</tbody>
</table>

8. 

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>7</th>
<th>9</th>
<th>13</th>
<th>17</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>34</td>
<td>37</td>
<td>38</td>
<td>37</td>
<td>36</td>
<td>32</td>
<td>26</td>
</tr>
</tbody>
</table>

9. 

<table>
<thead>
<tr>
<th>x</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>24</td>
<td>28</td>
<td>30</td>
<td>32</td>
<td>36</td>
<td>39</td>
<td>43</td>
<td>45</td>
</tr>
</tbody>
</table>

Estimate the correlation coefficient for the data.

10. 

11. 

12. 

In Exercises 13 and 14, use the table below which gives the numbers \( y \) (in thousands) of hairdressers and cosmetologists in the United States from 1995 through 2000, where \( x = 5 \) represents 1995.

<table>
<thead>
<tr>
<th>x</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>750</td>
<td>737</td>
<td>748</td>
<td>763</td>
<td>784</td>
<td>820</td>
</tr>
</tbody>
</table>

13. Draw a scatter plot of the data. Tell whether a *linear*, *quadratic*, or *exponential* function would best model the data.


In Exercises 15–17, you will need a measuring tape.

15. To the nearest quarter of an inch, measure the heights and hip heights (the top of the hip to the floor) of 10 people standing barefoot.

16. Draw a scatter plot of the data (height \( x \), hip height \( y \)). Tell whether a *linear*, *quadratic*, or *exponential* function would best model the data.

17. Find and graph an equation of the best-fitting model. Choose a height that is not one of the heights you measured. Use your model to predict the hip height of a person of that height.
Collecting Data

**GOAL** Use simulations and surveys to collect data; identify biased samples.

One way to collect data about people or objects is by performing a simulation. A simulation is an experiment that models a real-life situation.

**EXAMPLE 1** Performing a Simulation

A movie theater is giving away a prize card with every ticket purchase. Each card allows the holder to get one of six different concession stand items. If you are equally likely to receive each of the cards, how many tickets would you need to buy in order to receive at least one of each type of prize card?

**SOLUTION**

Perform a simulation by using a number cube. Let each number on the cube represent a different type of prize card.

1. Roll the number cube. Record the result in a tally chart.
2. Continue rolling and recording until you have at least one tally mark for each prize card.
3. Count the total number of tickets purchased.

The results suggest that you would need to purchase about 16 tickets. Repeating the simulation and combining the results by calculating the mean of the numbers of tickets purchased would give a more accurate estimate.

<table>
<thead>
<tr>
<th>Prize card</th>
<th>Number of tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
</tr>
<tr>
<td>3</td>
<td>III</td>
</tr>
<tr>
<td>4</td>
<td>III</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

Another way to collect data is by conducting a survey. A population is a group of people or objects that you want information about. When it is difficult to survey an entire population, a sample, or part of the population, is surveyed. An unbiased sample is representative of the population you want information about. A biased sample overrepresents or underrepresents part of the population.

**EXAMPLE 2** Identifying a Biased Sample

A survey is being conducted to decide on a yearbook theme. Students can choose from a sports theme or a music theme. Tell whether the given sample is biased or unbiased.

a. Students on the volleyball team  
b. Students in line in the cafeteria

**SOLUTION**

a. The sample is biased because students on the volleyball team are more likely to want a sports theme.

b. The sample is unbiased because a wide range of students will be surveyed.
In a simulation, such as the one in Example 1, the results are more accurate if you increase the number of times you perform the simulation. Similarly, as the size of a sample increases, the sample will more accurately represent the population.

Because a sample is an approximation of an entire population, the results of a survey may not be exact. A **margin of sampling error** is a percent that indicates an interval that is likely, but not certain, to contain the exact result.

### MARGIN OF SAMPLING ERROR

For a random sample of size \( n \), taken from a large population, the margin of sampling error \( S \) can be approximated by this formula:

\[
S \approx \frac{1}{\sqrt{n}}
\]

### EXAMPLE 3  Finding a Margin of Sampling Error

In a survey of 1600 voters, 51\% said they voted for candidate A.

- **a.** What is the margin of sampling error for the survey?
- **b.** Give an interval that is likely to contain the exact percent of all voters who voted for candidate A.

#### SOLUTION

**a.** Use the formula for the margin of sampling error.

\[
S = \frac{1}{\sqrt{n}} \quad \text{Write margin of sampling error formula.}
\]

\[
= \frac{1}{\sqrt{1600}} \quad \text{Substitute 1600 for } n.
\]

\[
= 0.025 \quad \text{Simplify.}
\]

The margin of sampling error is about 2.5\%.

**b.** To find the interval, take the percent of people in the sample who voted for candidate A, 51\%, and subtract and add the margin of sampling error, 2.5\%.

\[
51\% - 2.5\% = 48.5\% \quad 51\% + 2.5\% = 53.5\%
\]

It is likely that the exact percent of all voters who voted for candidate A is between 48.5\% and 53.5\%.
Using a Margin of Sampling Error

In a school survey, 18% of students named root beer as their favorite soda. If the margin of sampling error is 5%, how many students were surveyed?

SOLUTION

To answer the question, use the formula for the margin of sampling error.

\[
S = \frac{1}{\sqrt{n}} \quad \text{Write margin of sampling error formula.}
\]

\[
0.05 = \frac{1}{\sqrt{n}} \quad \text{Substitute 0.05 for } S.
\]

\[
0.05\sqrt{n} = 1 \quad \text{Cross multiply.}
\]

\[
n = 400 \quad \text{Solve for } n.
\]

There were about 400 students surveyed.

EXERCISES

In Exercises 1 and 2, refer to Example 1 on page 993.

1. Repeat the simulation in Example 1 an additional 9 times. Find the mean of the total number of tickets for the 10 simulation results.

2. How does your answer to Exercise 1 compare with the results of the simulation in Example 1? Which of the results do you think is a more accurate approximation of the number of tickets you would have to buy to receive at least one of each type of prize card? Explain.

A survey of people’s favorite animals is being conducted. Tell whether the sample is biased or unbiased. Explain your reasoning.

3. People at a dog show

4. People at a pet store

5. Every twentieth person listed in the phone book

A survey of students’ favorite school subjects is being conducted. Tell whether the sample is biased or unbiased. Explain your reasoning.

6. Students from the math club

7. Every fifth student that enters the school

8. Every other student in the French club

9. Suppose you want to conduct a survey to find out how many books the average student reads in one year. Describe a biased sample and an unbiased sample that you could survey.

10. Ask both samples that you named in Exercise 9 the following survey question: “How many books have you read in the past year?” Compare your results.
Find the margin of sampling error for a survey with the given sample size. Round your answer to the nearest tenth of a percent.

11. 330
12. 10,000
13. 575
14. 1000
15. 2250
16. 900

Find the smallest sample size required for the given margin of sampling error. Round your answer to the nearest whole number.

17. 4%
18. 6%
19. 3.5%
20. 2.8%
21. 5.2%
22. 1.5%

In Exercises 23–25, design and perform a simulation to answer the question. Perform the simulation at least 10 times. (Hint: You may want to consider using a coin, a number cube, or index cards in a paper bag.)

23. You are playing a game of chance in which you are equally likely to win or lose. About how many times would you have to play the game in order to win at least once and lose at least once?

24. A gumball machine contains pink, blue, and white gumballs. There are twice as many pink gumballs as blue gumballs, and three times as many blue gumballs as white gumballs. If gumballs are randomly dispensed from the machine, what is the experimental probability of getting a blue gumball?

25. A brand of cereal has one of five different colored toy cars in each box. There are equal numbers of the blue, yellow, silver, and green cars. There are twice as many red cars as there are blue cars. Assuming the cars are randomly placed in boxes of cereal, about how many boxes would you have to buy in order to obtain at least one of each kind of car?

In Exercises 26–28, a survey reported that 15%, or 315 students, prefer having gym class during the last period of the day.

26. How many students were surveyed?
27. What is the margin of sampling error? Round your answer to the nearest tenth of a percent.
28. Give an interval that is likely to contain the exact percent of all students who prefer to have gym class during the last period.

In Exercises 29–32, a survey reported that 235 of the 500 voters in a sample voted for candidate A and the rest voted for candidate B.

29. What percent of the voters in the sample voted for candidate A? What percent of the voters in the sample voted for candidate B?
30. What is the margin of sampling error? Round your answer to the nearest tenth of a percent.
31. For each candidate, give an interval that is likely to contain the exact percent of all voters who voted for the candidate.
32. Based on your answer from Exercise 31, can you determine which candidate won? Explain.
33. What happens to the margin of sampling error as sample size increases? Give an example to support your answer.
Displaying and Analyzing Data

**GOAL** Display data in appropriate graphs; describe the effect of outliers.

Different data displays emphasize different aspects of data. Think about what you want to emphasize about a set of data before you choose a display.

<table>
<thead>
<tr>
<th>Type of Display</th>
<th>How Data is Shown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle graph</td>
<td>Shows data as parts of a whole</td>
</tr>
<tr>
<td>Histogram and bar graph</td>
<td>Compares data in different categories</td>
</tr>
<tr>
<td>Line graph</td>
<td>Shows data that change over time</td>
</tr>
<tr>
<td>Box-and-whisker plot</td>
<td>Shows the spread of data</td>
</tr>
<tr>
<td>Stem-and-leaf plot</td>
<td>Shows how data are clustered</td>
</tr>
</tbody>
</table>

### Example 1 Choosing a Data Display

For parts (a) and (b), use the table below which gives the areas of the Great Lakes.

<table>
<thead>
<tr>
<th>Lake</th>
<th>Erie (mi²)</th>
<th>Huron (mi²)</th>
<th>Michigan (mi²)</th>
<th>Ontario (mi²)</th>
<th>Superior (mi²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>31,820</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lake Erie</td>
<td>9940</td>
<td>23,010</td>
<td>22,400</td>
<td>7540</td>
<td>31,820</td>
</tr>
</tbody>
</table>

**a.** Create a data display that compares the area of Lake Superior to the area of Lake Erie.

You can see that the area of Lake Superior is more than three times the area of Lake Erie.

**b.** Create a data display that compares the combined areas of Lake Huron and Lake Michigan to the total area of the Great Lakes.

You can see that combined, Lake Huron and Lake Michigan make up about half of the total area of the Great Lakes.
**Misleading Graphs**  The way that a graph is drawn can sometimes give a misleading impression of data. Watch out for things like broken scales, collapsed data categories, and the scaled dimensions of objects that represent categories of data.

**Example 2  Identifying Misleading Graphs**

Tell how the graph could potentially be misleading.

**a.**

<table>
<thead>
<tr>
<th>Type of music</th>
<th>Percent of sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical</td>
<td>7%</td>
</tr>
<tr>
<td>Country</td>
<td>16%</td>
</tr>
<tr>
<td>Jazz</td>
<td>5%</td>
</tr>
<tr>
<td>Oldies</td>
<td>5%</td>
</tr>
<tr>
<td>Pop</td>
<td>13%</td>
</tr>
<tr>
<td>Rap</td>
<td>15%</td>
</tr>
<tr>
<td>R&amp;B</td>
<td>12%</td>
</tr>
<tr>
<td>Rock</td>
<td>27%</td>
</tr>
</tbody>
</table>

**b.**

<table>
<thead>
<tr>
<th>Year</th>
<th>Population of Allentown, PA (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>110</td>
</tr>
<tr>
<td>1980</td>
<td>104</td>
</tr>
<tr>
<td>1990</td>
<td>105</td>
</tr>
<tr>
<td>2000</td>
<td>107</td>
</tr>
</tbody>
</table>

**Solution**

**a.** It appears as if rap music is the second highest in music sales, but this is not really the case. The “other” category hides the fact that country music makes up 16% of sales.

**b.** It appears as if Allentown, PA, had a huge drop in population between 1970 and 1980, but this is not really the case. The broken vertical scale causes the numbers on the scale to be more spread out, which causes the data values to look further apart.

**Measures of Central Tendency and Dispersion**  Measures of central tendency and dispersion can give misleading impressions of a data set if the data set contains one or more outliers. An **outlier** is a value that is much greater than or much less than most of the other numbers in a data set.
EXAMPLE 3  Examining the Effect of Outliers

A city’s high temperatures, in degrees Fahrenheit, during a 14-day period were 36, 37, 36, 34, 33, 30, 30, 32, 31, 31, 32, 32, 33, and 35.

a. Calculate the mean, median, mode, range, and standard deviation of the data.

b. On the 15th day, the temperature was 49°F. Calculate the new mean, median, mode, range, and standard deviation.

c. Of the mean, median, and mode, which measure of central tendency is affected the most by the additional temperature? the least?

d. What effect does an outlier have on range and standard deviation?

SOLUTION

a. Mean: \( \bar{x} = \frac{30 + 30 + \ldots + 37}{14} = \frac{462}{14} = 33 \)  
Median: 32  
Mode: 32  
Range: 7

Std. Dev.: \( \sigma = \sqrt{\frac{(30 - 33)^2 + (30 - 33)^2 + \ldots + (37 - 33)^2}{14}} = \sqrt{\frac{68}{14}} \approx 2.2 \)

b. Mean: \( \bar{x} = \frac{30 + 30 + \ldots + 49}{15} = \frac{511}{15} \approx 34.1 \)  
Median: 33  
Mode: 32  
Range: 19

Std. Dev.: \( \sigma = \sqrt{\frac{(30 - 34.1)^2 + (30 - 34.1)^2 + \ldots + (49 - 34.1)^2}{15}} = \sqrt{\frac{300.54}{15}} \approx 4.5 \)

c. The mean is affected the most by the additional temperature. The mode is affected the least by the additional temperature.

d. The range and the standard deviation increase with the addition of an outlier.

EXERCISES

1. The table below shows the number of farms in the United States from 1995 through 2000. Draw a data display that shows how the number of farms changed from year to year.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of farms (thousands)</td>
<td>2196</td>
<td>2191</td>
<td>2191</td>
<td>2191</td>
<td>2192</td>
<td>2172</td>
</tr>
</tbody>
</table>

2. The table below shows the cost of airmailing a 4-ounce letter from the United States to four different countries in 2000. Draw a data display that shows how the costs of mailing to Canada, Mexico, and Spain compare to the cost of mailing to Japan. The cost of mailing to which country is about half of the cost of mailing to Japan?

<table>
<thead>
<tr>
<th>Country</th>
<th>Canada</th>
<th>Mexico</th>
<th>Spain</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (dollars)</td>
<td>$1.35</td>
<td>$1.65</td>
<td>$3.20</td>
<td>$3.50</td>
</tr>
</tbody>
</table>
In Exercises 3 and 4, use the following prices (in dollars) of CDs in a sale bin at a music store.


3. Draw a data display that can be used to visually compare the number of CDs that cost between $1.00 and $5.99 to the number of CDs that cost between $16.00 and $20.99. How do these two categories compare?

4. Draw a data display that can be used to find the range of the upper half of the prices. Then find the range of the upper half of the prices.

Tell how the graph could potentially be misleading. Then redraw the graph so that it is not misleading.

In Exercises 7–10, use the following data:

29, 64, 22, 25, 3, 35, 29, 22, 29, 32, 7

7. Calculate the mean, median, mode, range, and standard deviation.

8. Which data values would you consider to be outliers? Explain.

9. Calculate the mean, median, mode, range, and standard deviation for the data without the outliers you identified in Exercise 8.

10. Of the mean, median, or mode, which measure of central tendency would you use to describe the original set of data? Why?

In Exercises 11–14, use the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>6</th>
<th>2</th>
<th>7</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>3</th>
<th>7</th>
<th>5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

11. Use a graphing calculator to draw a scatter plot of the data. Then use linear regression to find and graph an equation of the best-fitting line.

12. What are the outlier(s) of the data set? Remove the outlier(s) from the data set and redraw the scatter plot.

13. Find and graph an equation of the best-fitting line for the new data set.

14. Explain how removing the outlier(s) from the data set affected the equation of the best-fitting line.