

EXPLORING DATA
AND STATISTICS

14.5

Modeling with
Trigonometric Functions

GOAL 1 WRITING A TRIGONOMETRIC MODEL

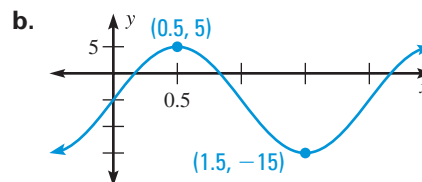
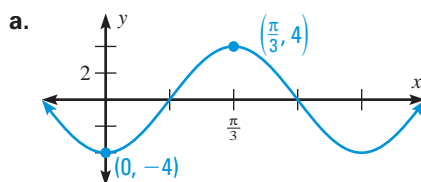
Graphs of sine and cosine functions are called *sinusoids*. When you write a sine or cosine function for a sinusoid, you need to find the values of a , $b > 0$, h , and k for

$$y = a \sin b(x - h) + k \quad \text{or} \quad y = a \cos b(x - h) + k$$

where $|a|$ is the amplitude, $\frac{2\pi}{b}$ is the period, h is the horizontal shift, and k is the vertical shift.

EXAMPLE 1 Writing Trigonometric Functions

Write a function for the sinusoid.



SOLUTION

- a. Since the maximum and minimum values of the function occur at points equidistant from the x -axis, the curve has no vertical shift. And because the minimum occurs on the y -axis, the graph is a reflection of a cosine curve with no horizontal shift. Therefore, the function has the form $y = a \cos bx$.

The period is $\frac{2\pi}{b} = \frac{2\pi}{3}$, so $b = 3$.

The amplitude is $|a| = 4$. Since the graph is a reflection, $a = -4$.

▶ The function is $y = -4 \cos 3x$.

- b. Since the maximum and minimum values of the function do *not* occur at points equidistant from the x -axis, the curve has a vertical shift. To find the value of k , add the maximum and minimum values and divide by 2:

$$k = \frac{M + m}{2} = \frac{5 + (-15)}{2} = \frac{-10}{2} = -5$$

Because the graph crosses the y -axis at $y = k$, the graph is a sine curve with no horizontal shift. Therefore, the function has the form $y = a \sin bx - 5$.

The period is $\frac{2\pi}{b} = 2$, so $b = \pi$.

The amplitude is $|a| = \frac{M - m}{2} = \frac{5 - (-15)}{2} = \frac{20}{2} = 10$.

Since the graph is not a reflection, $a = 10 > 0$.

▶ The function is $y = 10 \sin \pi x - 5$.

What you should learn

GOAL 1 Model data with a sine or cosine function.

GOAL 2 Use technology to write a trigonometric model, as applied in Example 4.

Why you should learn it

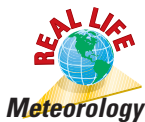
▼ To model many types of **real-life** quantities, such as the temperature inside and outside an igloo in Ex. 30.



STUDENT HELP

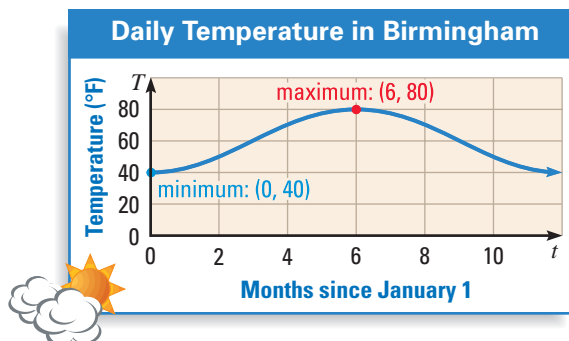
INTERNET HOMEWORK HELP

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EXAMPLE 2 Modeling a Sinusoid

Write a trigonometric model for the average daily temperature in Birmingham, Alabama. ▶ Source: National Climatic Data Center



SOLUTION

Notice that the graph crosses the T -axis at the minimum point. So if you model the temperature curve with a cosine function, there is a reflection but no horizontal shift. The mean of the maximum and minimum values is 60, so there is a vertical shift of $k = 60$.

The period is $\frac{2\pi}{b} = 12$, so $b = \frac{\pi}{6}$.

The amplitude is $|a| = 20$, and because the graph is a reflection it follows that $a = -20$.

▶ The model is $T = -20 \cos \frac{\pi}{6}t + 60$ where t is measured in months and $t = 0$ represents January 1.



EXAMPLE 3 Modeling Circular Motion

A Ferris wheel with a radius of 25 feet is rotating at a rate of 3 revolutions per minute. When $t = 0$, a chair starts at the lowest point on the wheel, which is 5 feet above the ground. Write a model for the height h (in feet) of the chair as a function of the time t (in seconds).

SOLUTION

When the chair is at the bottom of the Ferris wheel, it is 5 feet above the ground, so $m = 5$. When it is at the top, it is $5 + 2(25) = 55$ feet above the ground, so $M = 55$.

The vertical shift for the model is $k = \frac{M + m}{2} = \frac{55 + 5}{2} = \frac{60}{2} = 30$.

When $t = 0$, the height is at its minimum, so the model is a cosine function with $a < 0$ and no horizontal shift.

The amplitude is $|a| = \frac{M - m}{2} = \frac{55 - 5}{2} = \frac{50}{2} = 25$. Because $a < 0$ it follows that $a = -25$.

Since the Ferris wheel is rotating at 3 revolutions per minute, it completes one revolution in 20 seconds. The period is $\frac{2\pi}{b} = 20$, so $b = \frac{\pi}{10}$.

▶ A model for the height of the chair as a function of time is $h = -25 \cos \frac{\pi}{10}t + 30$.

GOAL 2 SINUSOIDAL MODELING USING TECHNOLOGY

There are two ways you can model a set of data points whose scatter plot appears sinusoidal. One way is to estimate the minimum and maximum values and use the technique shown in Example 2. Another way is to use a graphing calculator that has a sinusoidal regression feature. The advantage of the second method is that it uses all of the data points to find the model.



EXAMPLE 4 Using Sinusoidal Regression

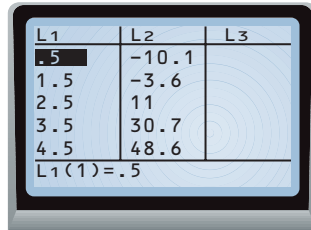
The average daily temperature T (in degrees Fahrenheit) in Fairbanks, Alaska, is given in the table. Time t is measured in months, with $t = 0$ representing January 1. Write a trigonometric model that gives T as a function of t .

t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
T	-10.1	-3.6	11.0	30.7	48.6	59.8	62.5	56.8	45.5	25.1	2.7	-6.5

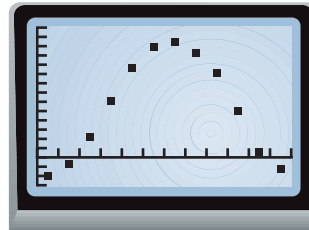
► Source: National Climatic Data Center

SOLUTION

Begin by entering the data in a graphing calculator and drawing a scatter plot.



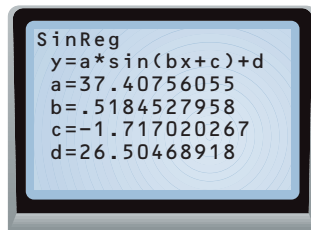
1 Enter the data.



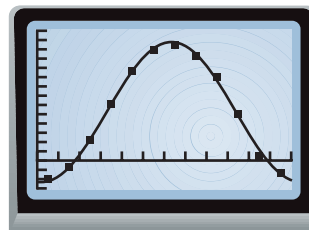
2 Draw a scatter plot.

Because the scatter plot appears sinusoidal, you can fit the data with a sine function to get the following model:

$$T = 37.4 \sin(0.518t - 1.72) + 26.5$$



3 Perform a sinusoidal regression.



4 Graph the model and the data in the same viewing window.

After obtaining the model, you should graph it in the same viewing window as the scatter plot to see how well the model fits the data. In this case, the model is a good fit.

✓ **CHECK** As a check on the reasonableness of the model, notice that the period is $\frac{2\pi}{0.518} \approx 12$, which is the number of months in a year.

GUIDED PRACTICE

Vocabulary Check ✓

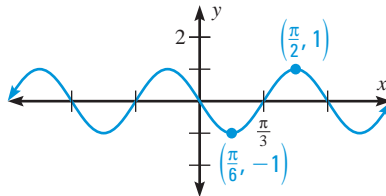
Concept Check ✓

- Complete this statement: Graphs of sine and cosine functions are called ?.
- Which two points are most useful when writing a sinusoidal model for a given graph or set of data? Explain.
- Describe a characteristic of a sinusoidal graph that you would model with a cosine function rather than a sine function.

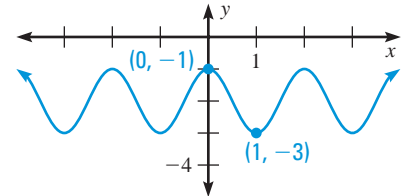
Skill Check ✓

Write a function for the sinusoid.

4.



5.



Write a function for the sinusoid with maximum at A and minimum at B.

- $A(0, 8)$, $B(\pi, -2)$
- $A(\pi, 10)$, $B(3\pi, 4)$
- $A(6, 5)$, $B(2, 1)$
- FERRIS WHEEL** Look back at Example 3. Suppose the Ferris wheel rotates at a rate of 4 revolutions per minute and has a radius of 20 feet. Write a model for the height h (in feet) of the chair as a function of the time t (in seconds).

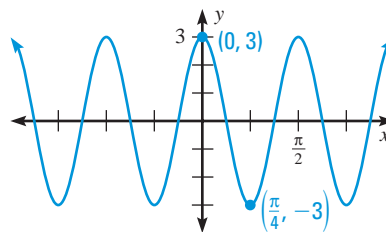
PRACTICE AND APPLICATIONS

STUDENT HELP

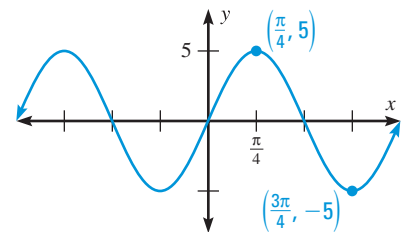
Extra Practice
to help you master
skills is on p. 960.

WRITING FUNCTIONS Write a function for the sinusoid.

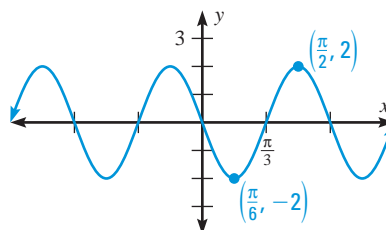
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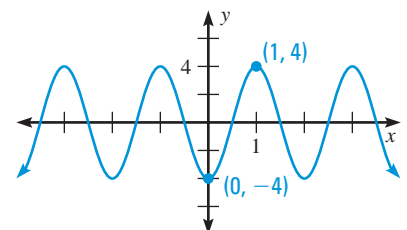
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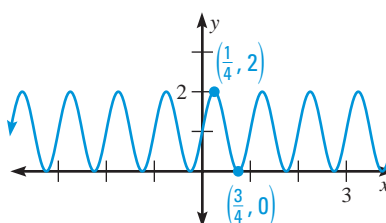
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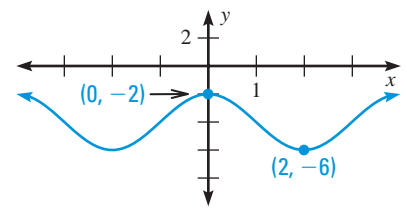
13.



14.



15.



STUDENT HELP

HOMEWORK HELP

- Example 1:** Exs. 10–27
Example 2: Exs. 30, 32
Example 3: Exs. 31, 33
Example 4: Exs. 34, 35

WRITING TRIGONOMETRIC FUNCTIONS Write a function for the sinusoid with maximum at **A** and minimum at **B**.

16. $A(0, 3)$, $B(2\pi, -3)$ 17. $A(\pi, 8)$, $B(3\pi, -8)$ 18. $A(9\pi, 1)$, $B(3\pi, -5)$
 19. $A\left(\frac{\pi}{3}, 4\right)$, $B(0, 2)$ 20. $A(2, 6)$, $B(6, 0)$ 21. $A(3, 7)$, $B(1, -3)$
 22. $A\left(\frac{2\pi}{3}, 9\right)$, $B(2\pi, 5)$ 23. $A\left(\frac{\pi}{6}, 11\right)$, $B(0, -1)$ 24. $A(0, 5)$, $B(4, -13)$
 25. $A(0, 0)$, $B(3\pi, -8)$ 26. $A(6, 2)$, $B(0, -4)$ 27. $A\left(\frac{\pi}{4}, -2\right)$, $B\left(\frac{\pi}{12}, -6\right)$

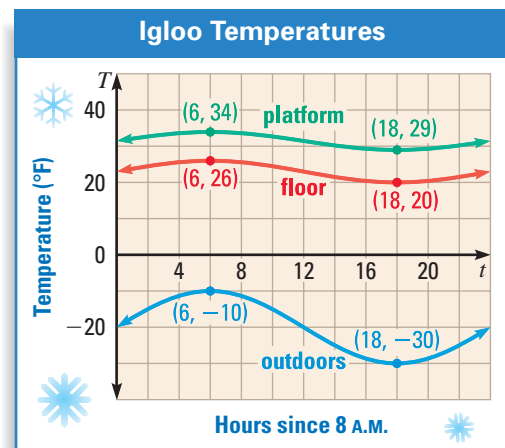
28. **CRITICAL THINKING** Any sinusoid can be modeled by either a sine function or a cosine function. Using the graph from part (a) of Example 1, find the values of a , b , h , and k for the model $y = a \sin b(x - h) + k$. Use identities to show that the model you found is equivalent to the model from part (a) of Example 1.

29. **Writing** Since any sinusoid can be modeled by either a sine function or a cosine function, you can choose the type of function that is more convenient. For a sinusoid whose y -intercept occurs halfway between the maximum and minimum values of the function, tell which type of function you would use to model the graph. Explain your answer.

30. **CLIMATE CONTROL**

Eskimos use igloos as temporary shelter from harsh winter weather. The graph shows the temperatures inside and outside an igloo throughout a typical winter day. Write a sinusoidal model for the outside temperature T (in degrees Fahrenheit) as a function of the time of day t (in hours since midnight). Then write sinusoidal models for the floor-level temperature and for the sleeping-platform temperature.

► Source: *Scientific American*



FOCUS ON APPLICATIONS



31. **STEAMSHIPS** The paddle wheel of the S.S. Beaver was 13 feet in diameter and revolved 30 times per minute when moving at top speed. Using this speed and starting from a point at the very top of the wheel, write a model for the height h (in feet) of the end of a paddle relative to the water's surface as a function of the time t (in minutes). (Assume the paddle is 2 feet below the water's surface at its lowest point.) ► Source: *S.S. Beaver: The Ship That Saved the West*

32. **OCEAN TIDES** The height of the water in a bay varies sinusoidally over time. On a certain day off the coast of Maine, a high tide of 10 feet occurred at 5:00 A.M. and a low tide of 2 feet occurred at 1:00 P.M. Write a model for the height h (in feet) of the water as a function of time t (in hours since midnight).

33. **SEWING MACHINES** In front of the Antique Sewing Machine Museum in Arlington, Texas, is the largest sewing machine in the world. The *flywheel*, which turns as the machine sews, is 5 feet in diameter. Write a model for the height h (in feet) of the handle on the flywheel as a function of the time t (in seconds), assuming that the wheel makes a complete revolution every 2 seconds and that the handle starts at its minimum height of 4 feet above the ground.



THE S.S. BEAVER was the first steamship on the Pacific Coast. It was built in the 1830s and sailed out of British Columbia, Canada, for over 50 years.

**STATISTICS CONNECTION**

In Exercises 34 and 35, use a graphing calculator to find a sinusoidal model that fits the data.

34. **METEOROLOGY** The average daily temperature T (in degrees Fahrenheit) in Moline, Illinois, is given in the table. Time t is measured in months, with $t = 0$ representing January 1. Find a model for the data. ▶ Source: National Climatic Data Center

t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
T	19.9	24.8	37.4	50.4	61.4	71.1	75.2	72.7	64.7	53.0	39.6	25.4

35. **HEATING DEGREE-DAYS** For any given day, the number of degrees that the average temperature is below 65°F is called the *degree-days* for that day. This figure is used to calculate how much is spent on heating. The table below gives the total number T of degree-days for each month t in Dubuque, Iowa, with $t = 1$ representing January. Find a model for the data. ▶ Source: *Workshop Math*

t	1	2	3	4	5	6	7	8	9	10	11	12
T	1420	1204	1026	546	260	78	12	31	156	450	906	1287

Test Preparation

36. **MULTIPLE CHOICE** During one cycle, a sinusoid has a minimum at $(18, 44)$ and a maximum at $(30, 68)$. What is the amplitude of this sinusoid?
- (A) 6 (B) 12 (C) 22 (D) 24 (E) 48
37. **MULTIPLE CHOICE** During one cycle, a sinusoid has a maximum at $(4, 12)$ and a minimum at $(12, -2)$. What is the period of this sinusoid?
- (A) 8 (B) 8π (C) 16 (D) 16π (E) 32

★ Challenge

38. **FINDING A FUNCTION** Write a sinusoidal function whose graph has a minimum at $(\pi, 4)$ and a maximum at $(\frac{\pi}{2}, 7)$.

MIXED REVIEW

FINDING PROBABILITY Two six-sided dice are rolled. Find the probability of the given event. (Review 12.4)

39. The sum is 3. 40. The sum is 6. 41. The sum is 12.
42. The sum is odd. 43. The sum is 5 or 6. 44. The sum is less than 7.

EVALUATING FUNCTIONS Evaluate the function without using a calculator. (Review 13.3 for 14.6)

45. $\cos(-225^\circ)$ 46. $\cos 240^\circ$ 47. $\sin(-30^\circ)$
48. $\tan 330^\circ$ 49. $\tan\left(-\frac{7\pi}{6}\right)$ 50. $\sin\frac{5\pi}{4}$

FINDING AREA Find the area of $\triangle ABC$ having the given side lengths. (Review 13.6)

51. $a = 3, b = 8, c = 10$ 52. $a = 12, b = 20, c = 25$
53. $a = 5, b = 9, c = 11$ 54. $a = 4, b = 6, c = 7$
55. $a = 6, b = 11, c = 14$ 56. $a = 8, b = 13, c = 20$

QUIZ 2

Self-Test for Lessons 14.3–14.5

Simplify the expression. (Lesson 14.3)


$$1. \tan\left(\frac{\pi}{2} - x\right) \sec x \quad 2. 1 - \sin^2 x + \frac{\cos x}{\sec x} \quad 3. \cos(-x) \cos\left(\frac{\pi}{2} - x\right)$$

Find the general solution of the equation. (Lesson 14.4)

$$4. 4 \cos^2 x - 3 = 0 \quad 5. 3 \sin^2 x - 8 \sin x = 3 \quad 6. \sqrt{3} \tan^2 x + 4 \tan x = -\sqrt{3}$$

Write a function for the sinusoid with maximum at A and minimum at B . (Lesson 14.5)

$$7. A\left(\frac{3\pi}{4}, 5\right), B\left(\frac{\pi}{4}, -5\right) \quad 8. A(0, 3), B(3\pi, 1) \quad 9. A(\pi, 6), B(0, 2)$$

10.  **METEOROLOGY** The average daily temperature T (in degrees Fahrenheit) in Detroit, Michigan, is given in the table. Time t is measured in months, with $t = 0$ representing January 1. Find a model for the data. (Lesson 14.5)

t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
T	22.9	25.4	35.7	47.3	58.4	67.6	72.3	70.5	63.2	51.2	40.2	28.3

MATH & History

Music and Math




APPLICATION LINK
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THEN

MUSIC AND MATH have been studied together for many centuries. For instance, 2500 years ago Pythagoras discovered that when the ratio of the lengths of two strings is a whole number, plucking the strings produces harmonious tones.

NOW

TODAY we know that musical notes can be modeled by sine functions. The function $y = \sin 2\pi fx$ models a note with frequency f (in hertz) where x represents the length of time (in seconds) that the note is played. By adding the functions modeling two different notes, you can analyze the sound that occurs when the notes are played together.

- Write functions that model notes with frequencies 10 hertz, 11 hertz, 12 hertz, 13 hertz, 14 hertz, and 15 hertz.
-  Use a graphing calculator to graph the sum of the models for 10 hertz and 15 hertz. How many cycles of the function occur in 1 second?
- How many cycles per second are there in the function that describes what you hear when notes with frequencies of 10 and 14 hertz are played together? 10 and 13 hertz? 10 and 12 hertz?



The lyre is introduced.

c. 3000 B.C.

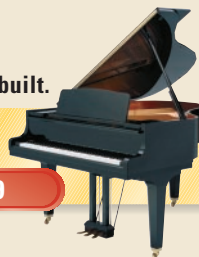
Harp first appeared.

c. A.D. 700



First piano built.

1709



1964



Synthesizer invented.