EXPLORING DATA AND STATISTICS

1.5

Problem Solving Using Algebraic Models

What you should learn

GOAL 1 Use a general problem solving plan to solve **real-life** problems, as in **Example 2**.

GOAL 2 Use other problem solving strategies to help solve **real-life** problems, as in **Ex. 22**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the average speed of the Japanese Bullet Train in **Example 1**.

REAL LIFE

GOAL 1 USING A PROBLEM SOLVING PLAN

One of your major goals in this course is to learn how to use algebra to solve real-life problems. You have solved simple problems in previous lessons, and this lesson will provide you with more experience in problem solving.

As you have seen, it is helpful when solving real-life problems to first write an equation in words *before* you write it in mathematical symbols. This word equation is called a **verbal model**. The verbal model is then used to write a mathematical statement, which is called an **algebraic model**. The key steps in this problem solving plan are shown below.

Write a verbal model. → **Assign labels.** → **Write an algebraic model.** → **Solve the algebraic model.** → **Answer the question.**

EXAMPLE 1 *Writing and Using a Formula*

The Bullet Train runs between the Japanese cities of Osaka and Fukuoka, a distance of 550 kilometers. When it makes no stops, it takes 2 hours and 15 minutes to make the trip. What is the average speed of the Bullet Train?

SOLUTION

You can use the formula $d = rt$ to write a verbal model.

VERBAL MODEL: Distance = Rate · Time

LABELS:
Distance = **550** (kilometers)
Rate = r (kilometers per hour)
Time = **2.25** (hours)

ALGEBRAIC MODEL:

$550 = r(2.25)$ **Write algebraic model.**

$\frac{550}{2.25} = r$ **Divide each side by 2.25.**

$244 \approx r$ **Use a calculator.**

▶ The Bullet Train's average speed is about 244 kilometers per hour.

UNIT ANALYSIS You can use unit analysis to check your verbal model.

$$550 \text{ kilometers} \approx \frac{244 \text{ kilometers}}{\text{hour}} \cdot 2.25 \text{ hours}$$

REAL LIFE
Water Conservation

EXAMPLE 2 *Writing and Using a Simple Model*

A water-saving faucet has a flow rate of at most 9.6 cubic inches per second. To test whether your faucet meets this standard, you time how long it takes the faucet to fill a 470 cubic inch pot, obtaining a time of 35 seconds. Find your faucet's flow rate. Does it meet the standard for water conservation?

PROBLEM SOLVING STRATEGY

SOLUTION

VERBAL MODEL

Volume of pot = Flow rate of faucet · Time to fill pot

LABELS

Volume of pot = **470** (cubic inches)

Flow rate of faucet = ***r*** (cubic inches per second)

Time to fill pot = **35** (seconds)

ALGEBRAIC MODEL

$470 = r(35)$ **Write algebraic model.**

$13.4 \approx r$ **Divide each side by 35.**

▶ The flow rate is about 13.4 in.^{3}/sec, which does not meet the standard.

REAL LIFE
Gasoline Cost

EXAMPLE 3 *Writing and Using a Model*

You own a lawn care business. You want to know how much money you spend on gasoline to travel to out-of-town clients. In a typical week you drive 600 miles and use 40 gallons of gasoline. Gasoline costs $1.25 per gallon, and your truck's fuel efficiency is 21 miles per gallon on the highway and 13 miles per gallon in town.

SOLUTION

VERBAL MODEL

Total miles = [highway miles] Fuel efficiency · Amount of gasoline + [local miles] Fuel efficiency · Amount of gasoline

LABELS

Total miles = **600** (miles)

Fuel efficiency (highway) = **21** (miles per gallon)

Amount of gasoline (highway) = ***x*** (gallons)

Fuel efficiency (local) = **13** (miles per gallon)

Amount of gasoline (local) = **40 − *x*** (gallons)

ALGEBRAIC MODEL

$600 = 21x + 13(40 - x)$ **Write algebraic model.**

$600 = 8x + 520$ **Simplify.**

$80 = 8x$ **Subtract 520 from each side.**

$10 = x$ **Divide each side by 8.**

▶ In a typical week you use 10 gallons of gasoline to travel to out-of-town clients. The cost of the gasoline is (10 gallons)($1.25 per gallon) = $12.50.

STUDENT HELP

Study Tip
The solutions of the equations in Examples 2 and 3 are 13.4 and 10, respectively. However, these are not the answers to the questions asked. In Example 2 you must compare 13.4 to 9.6, and in Example 3 you must multiply 10 by $1.25. Be certain to answer the question asked.

FOCUS ON APPLICATIONS

RAILROADS In 1862, two companies were given the rights to build a railroad from Omaha, Nebraska to Sacramento, California. The Central Pacific Company began from Sacramento in 1863. Twenty-four months later, the Union Pacific Company began from Omaha.

INTERNET APPLICATION LINK
www.mcdougallittell.com

GOAL 2 USING OTHER PROBLEM SOLVING STRATEGIES

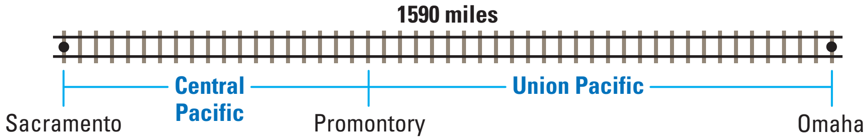
When you are writing a verbal model to represent a real-life problem, remember that you can use other problem solving strategies, such as *draw a diagram, look for a pattern*, or *guess, check, and revise,* to help create the verbal model.

EXAMPLE 4 *Drawing a Diagram*

RAILROADS Use the information under the photo at the left. The Central Pacific Company averaged 8.75 miles of track per month. The Union Pacific Company averaged 20 miles of track per month. The photo shows the two companies meeting in Promontory, Utah, as the 1590 miles of track were completed. When was the photo taken? How many miles of track did each company build?

SOLUTION

Begin by drawing and labeling a diagram, as shown below.



VERBAL MODEL

Total miles of track = [Central Pacific] Miles per month · Number of months + [Union Pacific] Miles per month · Number of months

LABELS

Total miles of track = **1590** (miles)

Central Pacific rate = **8.75** (miles per month)

Central Pacific time = ***t*** (months)

Union Pacific rate = **20** (miles per month)

Union Pacific time = ***t* − 24** (months)

ALGEBRAIC MODEL

$1590 = 8.75t + 20(t - 24)$ **Write algebraic model.**

$1590 = 8.75t + 20t - 480$ **Distributive property**

$2070 = 28.75t$ **Simplify.**

$72 = t$ **Divide each side by 28.75.**

▶ The construction took 72 months (6 years) from the time the Central Pacific Company began in 1863. So, the photo was taken in 1869. The number of miles of track built by each company is as follows.

Central Pacific: $\frac{8.75 \text{ miles}}{\text{month}} \cdot 72 \text{ months} = 630 \text{ miles}$

Union Pacific: $\frac{20 \text{ miles}}{\text{month}} \cdot (72 - 24) \text{ months} = 960 \text{ miles}$

STUDENT HELP

Skills Review
For help with additional problem solving strategies, see p. 930.

Heights

EXAMPLE 5 *Looking for a Pattern*

The table gives the heights to the top of the first few stories of a tall building. Determine the height to the top of the 15th story.

Story	Lobby	1	2	3	4
Height to top of story (feet)	20	32	44	56	68

SOLUTION

Look at the differences in the heights given in the table. After the lobby, the height increases by 12 feet per story.

Heights: 20 (+12) 32 (+12) 44 (+12) 56 (+12) 68

You can use the observed pattern to write a model for the height.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Height to top of a story = **Height of lobby** + **Height per story** · **Story number**

LABELS

Height to top of a story = h (feet)

Height of lobby = **20** (feet)

Height per story = **12** (feet per story)

Story number = n (stories)

ALGEBRAIC MODEL

$h = 20 + 12n$ **Write algebraic model.**

$= 20 + 12(15)$ **Substitute 15 for *n*.**

$= 200$ **Simplify.**

▶ The height to the top of the 15th story is 200 feet.

EXAMPLE 6 *Guess, Check, and Revise*

WEATHER BALLOONS A spherical weather balloon needs to hold 175 cubic feet of helium to be buoyant enough to lift an instrument package to a desired height. To the nearest tenth of an foot, what is the radius of the balloon?

SOLUTION

Use the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$.

$175 = \frac{4}{3}\pi r^3$ **Substitute 175 for *V*.**

$42 \approx r^3$ **Divide each side by $\frac{4}{3}\pi$.**

You need to find a number whose cube is 42. As a first guess, try $r = 4$. This gives $4^3 = 64$. Because $64 > 42$, your guess of 4 is too high. As a second guess, try $r = 3.5$. This gives $(3.5)^3 = 42.875$, and $42.875 \approx 42$. So, the balloon's radius is about 3.5 feet.

FOCUS ON APPLICATIONS

WEATHER BALLOONS

Hundreds of weather balloons are launched daily from weather stations. The balloons typically carry about 40 pounds of instruments. Balloons usually reach an altitude of about 90,000 feet.

GUIDED PRACTICE

Vocabulary Check ✓

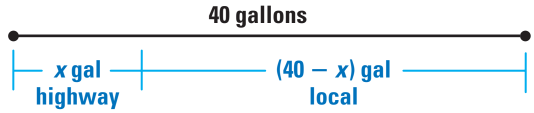
1. What is a verbal model? What is it used for?

Concept Check ✓

2. Describe the steps of the problem solving plan.

Skill Check ✓

3. How does this diagram help you set up the algebraic model in Example 3?



SCIENCE CONNECTION **In Exercises 4–7, use the following information.** To study life in Arctic waters, scientists worked in an underwater building called a *Sub-Igloo* in Resolute Bay, Canada. The water pressure at the floor of the Sub-Igloo was 2184 pounds per square foot. Water pressure is zero at the water's surface and increases by 62.4 pounds per square foot for each foot of depth.

4. Write a verbal model for the water pressure.
5. Assign labels to the parts of the verbal model. Indicate the units of measure.
6. Use the labels to translate the verbal model into an algebraic model.
7. Solve the algebraic model to find the depth of the Sub-Igloo's floor.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 940.

BOAT TRIP **In Exercises 8–11, use the following information.** You are on a boat on the Seine River in France. The boat's speed is 32 kilometers per hour. The Seine has a length of 764 kilometers, but only 547 kilometers can be navigated by boats. How long will your boat ride take if you travel the entire navigable portion of the Seine? Use the following verbal model.

$$\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$$

8. Assign labels to the parts of the verbal model.
9. Use the labels to translate the verbal model into an algebraic model.
10. Solve the algebraic model.
11. Answer the question.

MUSIC **In Exercises 12–14, use the following information.** A *metronome* is a device similar to a clock and is used to maintain the tempo of a musical piece. Suppose one particular piece has 180 measures with 3 beats per measure and a metronome marking of 80 beats per minute. Determine the length (in minutes) of the musical piece by using the following verbal model.

$$\boxed{\begin{matrix}\text{Metronome}\\\text{marking}\end{matrix}} \cdot \boxed{\begin{matrix}\text{Length of}\\\text{musical piece}\end{matrix}} = \boxed{\begin{matrix}\text{Number of measures}\\\text{in musical piece}\end{matrix}} \cdot \boxed{\begin{matrix}\text{Number of beats}\\\text{per measure}\end{matrix}}$$

STUDENT HELP

HOMEWORK HELP
Examples 1–3: Exs. 8–17
Examples 4–6: Exs. 18–27

12. Assign labels to the parts of the verbal model.
13. Use the labels to translate the verbal model into an algebraic model.
14. Answer the question. Use unit analysis to check your answer.

FOCUS ON APPLICATIONS

REAL LIFE **THE CHUNNEL** A tunnel under the English Channel was an engineering possibility for over a century before its completion. High speed trains traveling up to 300 kilometers per hour link London, England to Paris, France and Brussels, Belgium.

INTERNET APPLICATION LINK
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CALORIE INTAKE In Exercises 15–17, use the following information.
To determine the total number of calories of a food, you must add the number of calories provided by the grams of fat, the grams of protein, and the grams of carbohydrates. There are 9 Calories per gram of fat. A gram of protein and a gram of carbohydrates each have about 4 Calories. ▶ Source: U.S. Department of Agriculture

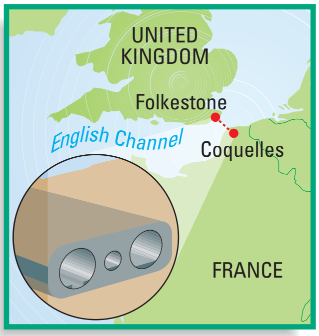
15. Write a verbal model that gives the total number of calories of a certain food.

16. Assign labels to the parts of the verbal model. Use the labels to translate the verbal model into an algebraic model.

17. One cup of raisins has 529.9 Calories and contains 0.3 gram of fat and 127.7 grams of carbohydrates. Solve the algebraic model to find the number of grams of protein in the raisins. Use unit analysis to check your answer.

18. **BORROWING MONEY** You have borrowed \$529 from your parents to buy a mountain bike. Your parents are not charging you interest, but they want to be repaid as soon as possible. You can afford to repay them \$20 per week. How long will it take you to repay your parents?

19. **THE CHUNNEL** The Chunnel connects the United Kingdom and France by a railway tunnel under the English Channel. The British started tunneling 2.5 months before the French and averaged 0.63 kilometer per month. The French averaged 0.47 kilometer per month. When the two sides met, they had tunneled 37.9 kilometers. How many kilometers of tunnel did each country build? If the French started tunneling on February 28, 1988, approximately when did the two sides meet?



20. **FLYING LESSONS** You are taking flying lessons to get a private pilot's license. The cost of the introductory lesson is $\frac{5}{8}$ the cost of each additional lesson, which is \$80. You have a total of \$375 to spend on the flying lessons. How many lessons can you afford? How much money will you have left?

21. **TYPING PAPERS** Some of your classmates ask you to type their history papers throughout a 7 week summer course. How much should you charge per page if you want to earn enough to pay for the flying lessons in Exercise 20 and have \$75 left over for spending money? You estimate that you can type 40 pages per week. Assume that you have to take 9 flying lessons plus the introductory lesson and that you already have \$375 to spend on the lessons.

22. **WOODSHOP** You are working on a project in woodshop. You have a wooden rod that is 72 inches long. You need to cut the rod so that one piece is 6 inches longer than the other piece. How long should each piece be?

23. **GARDENING** You have 480 feet of fencing to enclose a rectangular garden. You want the length of the garden to be 30 feet greater than the width. Find the length and width of the garden if you use all of the fencing.

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 24.

24. **WINDOW DISPLAYS** You are creating a window display at a toy store using wooden blocks. The display involves stacking blocks in triangular forms. You begin the display with 1 block, which is your first "triangle," and then stack 3 blocks, two on the bottom and one on the top, to get the next triangle. You create the next three triangles by stacking 6 blocks, then 10 blocks, and then 15 blocks. How many blocks will you need for the ninth triangle?

SCIENCE CONNECTION **In Exercises 25–27, use the following information.** As part of a science experiment, you drop a ball from various heights and measure how high it bounces on the first bounce. The results of six drops are given below.

Drop height (m)	0.5	1.5	2	2.5	4	5
First bounce height (m)	0.38	1.15	1.44	1.90	2.88	3.85

25. How high will the ball bounce if you drop it from a height of 6 meters?

26. To continue the experiment, you must find the number of bounces the ball will make before it bounces less than a given number of meters. Your experiment shows the ball's bounce height is always the same percent of the height from which it fell before the bounce. Find the average percent that the ball bounces each time.

27. Find the number of times the ball bounces before it bounces less than 1 meter if it is dropped from a height of 3 meters.

Test Preparation

28. **MULTIPLE CHOICE** You work at a clothing store earning \$7.50 per hour. At the end of the year, you figure out that on a weekly basis you averaged 5 hours of overtime for which you were paid time and a half. How much did you make for the entire year? (Assume that a regular workweek is 40 hours.)

(A) \$15,600 (B) \$17,550 (C) \$18,000 (D) \$18,525 (E) \$19,500

29. **MULTIPLE CHOICE** You are taking piano lessons. The cost of the first lesson is one and one half times the cost of each additional lesson. You spend \$260 for six lessons. How much did the first lesson cost?

(A) \$52 (B) \$40 (C) \$43.33 (D) \$60 (E) \$34.67

★ Challenge

30. **OWNING A BUSINESS** You have started a business making papier-mâché sculptures. The cost to make a sculpture is \$.75. Your sculptures sell for \$14.50 each at a craft store. You receive 50% of the selling price. Each sculpture takes about 2 hours to complete. If you spend 14 hours per week making sculptures, about how many weeks will you work to earn a profit of \$360?

EXTRA CHALLENGE
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MIXED REVIEW

LOGICAL REASONING **Tell whether the compound statement is *true* or *false*.** (Skills Review, p. 924)

31. $-3 < 5$ and $-3 > -5$

32. $1 > -2$ or $1 < -2$

33. $-4 > -5$ and $1 < -2$

34. $-2.7 > -2.5$ or $156 > 165$

ORDERING NUMBERS **Write the numbers in increasing order.** (Review 1.1)

35. $-1, -5, 4, -10, -55$

36. $-\frac{2}{3}, \frac{5}{8}, \frac{1}{100}, -2, 1$

37. $-1.2, 2, -2.9, 2.09, -2.1$

38. $-\sqrt{3}, 1, \sqrt{10}, \sqrt{2}, \frac{8}{5}$

SOLVING EQUATIONS **Solve the equation. Check your solution.** (Review 1.3 for 1.6)

39. $6x + 5 = 17$

40. $5x - 4 = 7x + 12$

41. $2(3x - 1) = 5 - (x + 3)$

42. $\frac{2}{3}x + \frac{1}{4} = 2x - \frac{5}{6}$

QUIZ 2

Self-Test for Lessons 1.3–1.5

Solve the equation. Check your solution. (Lesson 1.3)

1. $5x - 9 = 11$

2. $6y + 8 = 3y - 16$

3. $\frac{1}{4}z + \frac{2}{3} = \frac{1}{2}z - \frac{3}{4}$

4. $0.4(x - 50) = 0.2x + 12$

Solve the equation for *y*. Then find the value of *y* when *x* = 2. (Lesson 1.4)

5. $3x + 5y = 9$

6. $4x - 3y = 14$

7. The formula for the area of a rhombus is $A = \frac{1}{2}d_1d_2$ where d_1 and d_2 are the lengths of the diagonals. Solve the formula for d_1. (Lesson 1.4)

8. **GIRL SCOUT COOKIES** Your sister is selling Girl Scout cookies that cost $2.80 per box. Your family bought 6 boxes. How many more boxes of cookies must your sister sell in order to collect $154? (Lesson 1.5)

MATH & History

Problem Solving

APPLICATION LINK
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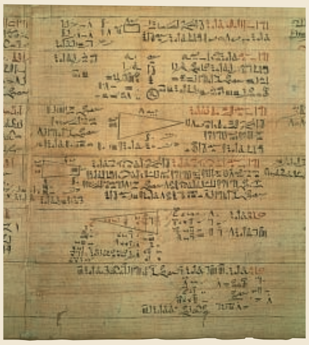
THEN

MANY CULTURES, such as the Egyptians, Greeks, Hindus, and Arabs, solved problems by using the *rule of false position*. This technique was similar to the problem solving strategy of guess, check, and revise. As an example of how to use the rule of false position, consider this problem taken from the Ahmes papyrus:

You want to divide 700 loaves of bread among four people in the ratio $\frac{2}{3}:\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. *Choose a number divisible by the denominators 2, 3, and 4, such as 48. Then evaluate* $\frac{2}{3}(48) + \frac{1}{2}(48) + \frac{1}{3}(48) + \frac{1}{4}(48)$, *which has a value of 84.*



Ahmes papyrus

1. The next step is to multiply 48 by a number so that when the resulting product is substituted for 48 in the expression above, you get a new expression whose value is 700. By what number should you multiply 48? How did you use the original expression's value of 84 to get your answer?

2. Use your result from Exercise 1 to find the number of loaves for each person.

NOW

TODAY, we would model this problem using $\frac{2}{3}x + \frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = 700$. Equations like this can now be solved with symbolic manipulation software.