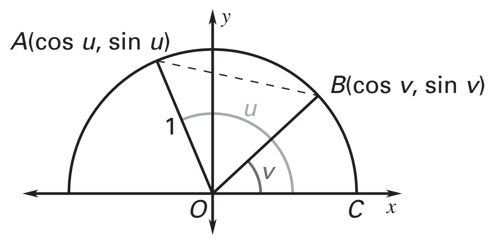
NAME ______________________ DATE __________

Challenge: Skills and Applications

For use with pages 869–874

1. Simplify the expression $\sin 4x \cos 3x - \cos 4x \sin 3x$.
2. In this exercise, you will derive the formula for $\cos(u - v)$.
   - a. In the diagram, what is the measure of $\angle AOB$, in terms of u and v?
   - b. Use the law of cosines to find $(AB)^2$, in terms of u and v.
   - c. Calculate $(AB)^2$, in terms of u and v, using the distance formula.
   - d. By setting equal the two expressions for $(AB)^2$ that you found in parts (b) and (c), prove the formula for $\cos(u - v)$.



3. In this exercise, you will write $A \sin u + B \cos u$ as the sine of a sum.
   - a. Show that $P\left(\frac{A}{\sqrt{A^2 + B^2}}, \frac{B}{\sqrt{A^2 + B^2}}\right)$ (with A and B not both 0) is a point on the unit circle. (*Hint:* Show that this point satisfies the equation of the circle.)
   - b. Let v be the angle formed by the radius drawn to P and the positive x-axis. Write the coordinates of P as trigonometric functions of v.
   - c. Write $A \sin u + B \cos u$ as $\sqrt{A^2 + B^2}\left(\frac{A}{\sqrt{A^2 + B^2}} \sin u + \frac{B}{\sqrt{A^2 + B^2}} \cos u\right)$. Rewrite this expression using the sum formula for sine.
4. Sometimes it is helpful to write the product $\sin u \cos v$ as a sum.
   - a. Add the formulas for $\sin(u + v)$ and $\sin(u - v)$ and solve for $\sin u \cos v$.
   - b. By adding the formulas for $\cos(u + v)$ and $\cos(u - v)$, find a similar formula that converts the product $\cos u \cos v$ into a sum.
   - c. How do you think you might be able to write $\sin u \sin v$ as a sum? Find a formula for this product.