

Challenge: Skills and Applications

For use with pages 848–854

In Exercises 1 and 2, write each expression in terms of a single trigonometric function.

$$1. \frac{\tan x}{1 - \cos x} - \frac{\tan x}{1 + \cos x}$$

$$2. \frac{\sec x}{\cos x} - \frac{\sec x - \cos x}{\sin x \tan x}$$

In Exercises 3–11, verify each identity.

$$3. \frac{\cos x}{1 - \sin x} = \sec x + \tan x$$

$$4. \tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$$

$$5. \sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$$

$$6. \cos^4 x - \sin^4 x = 1 - 2 \sin^2 x$$

$$7. (\tan x + \cot x)^2 = \csc^2 x \sec^2 x$$

$$8. \frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x} = 2 \tan^2 x$$

$$9. \frac{\sec x - 1}{\tan x} + \frac{\tan x}{\sec x + 1} = \frac{2 \sin x}{1 + \cos x}$$

$$10. \sec x(\cos x + \sin x)^2 = \sec x + 2 \sin x$$

$$11. \frac{\sec x - \tan x}{\cos x} - \frac{\cos x}{\sec x + \tan x} = \frac{\sin^2 x}{1 + \sin x}$$

12. Verify the following identity by simplifying each side separately to the same expression.

$$\sec x(\sec x - \cos x) = (\sec x \csc x - \cot x)^2$$

13. Suppose $x = \cos t$ and $y = 2 + \sin^2 t$ are parametric equations, for $0 \leq t \leq 2\pi$.

- State the range of x and the range of y .
- Using a trigonometric identity, rewrite the expression for y in terms of $\cos t$.
- Using substitution, eliminate the parameter t .
- Sketch the graph defined by the parametric equations. Identify the type of graph defined.