

Challenge: Skills and Applications

For use with pages 792–798

In Exercises 1–3, find each value. Use the following method: Draw a right triangle containing the angle represented by the inverse trigonometric function. Label the sides with their lengths. Use these to find the desired value.

1. $\sin\left(\cos^{-1}\frac{3}{5}\right)$

2. $\tan\left(\sin^{-1}\frac{7}{25}\right)$

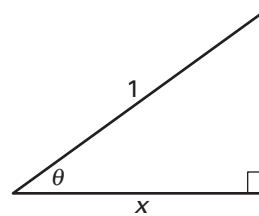
3. $\cos\left(\tan^{-1}\frac{5}{12}\right)$

In Exercises 4 and 5, write each expression as an algebraic function (i.e., without using trigonometric functions) for $0 \leq x \leq 1$.

Example $\sin(\cos^{-1}x)$

Solution Draw a triangle with one leg of length x and hypotenuse 1, as shown. Then $\theta = \cos^{-1}x$. By the Pythagorean theorem, the length of the other leg is

$$\sqrt{1-x^2}. \text{ Thus, } \sin(\cos^{-1}x) = \frac{\text{opp}}{\text{hyp}} = \sqrt{1-x^2}.$$



4. $\cos(\tan^{-1}x)$

5. $\tan(\sin^{-1}x)$

6. $\tan(\cos^{-1}x)$

7. a. Use a calculator, in degree mode, to find the value of $\sin^{-1}x + \cos^{-1}x$ for several values of x between 0 and 1. Try the same experiment with your calculator in radian mode. What do you notice?
- b. Using a right triangle diagram, justify your observations in part (a).
8. Tell whether each of the following statements is true for all x , $0 \leq x \leq 1$. If the statement is not true, correct it (without changing the left-hand side).
- a. $\sin^{-1}(-x) = -\sin^{-1}x$ b. $\cos^{-1}(-x) = -\cos^{-1}x$
9. The line $3y + x = 10$ intersects the circle $x^2 + y^2 = 50$ in two distinct points.
- a. Find the coordinates of the two points of intersection.
- b. Suppose you drew a radius to each of the two points of intersection you found in part (a). Find the angle θ , in standard position with $0^\circ \leq \theta < 360^\circ$, created by each radius. Give your answers to the nearest hundredth of a degree.
- c. Find the (smaller) angle between the two radii referred to in part (b) to the nearest hundredth of a degree.