NAME ______________________ DATE ________

Challenge: Skills and Applications

For use with pages 562–567

1. a. Write a function whose value for each number x is the sum of x and its reciprocal. Find the turning points of this function by graphing.

b. For what positive number x is the sum of x and its reciprocal a minimum? Does this quantity have a maximum value? Explain why or why not.

c. For what positive number x is the sum of x and *the square* of its reciprocal a minimum? (If you don't recognize this number, try cubing it.)

In Exercises 2–7, simplify the expression.

2. $\dfrac{(x+a)(x^{-1}-a^{-1})}{(x-a)(x^{-1}+a^{-1})}$

3. $\dfrac{\frac{1}{a}+\frac{1}{x}}{\frac{x}{a}-\frac{a}{x}}$

4. $\dfrac{b^{-2}-x^{-2}}{b^{-1}+x^{-1}}$

5. $\left(\dfrac{x}{b}-\dfrac{b}{x}\right) \div \left(1-\dfrac{b}{x}\right)$

6. $\dfrac{x}{x^2-4}+\dfrac{2}{x^2-2x}-\dfrac{x+1}{x^2+2x}$

7. $\left(x-a+\dfrac{2a^2}{x+a}\right)\left(\dfrac{1}{x^2}-\dfrac{1}{a^2}\right) \div \left(x^3-\dfrac{ax^3+a^4}{x+a}\right)$

8. a. Show, by finding A and B in terms of p and q, that if p and q are given, with $p \neq q$, then there are unique values A and B that make the following equation true.

$$\frac{1}{(x-p)(x-q)} = \frac{A}{x-p} + \frac{B}{x-q}$$

This is called the *partial-fraction decomposition* of the left-hand side. (*Hint*: After the right side is simplified, the numerators must be equal as polynomials.)

b. Use your answer to part (a) to find the partial-fraction decomposition of $\dfrac{1}{x^2-x-6}$.