

Challenge: Skills and Applications

For use with pages 474–479

In Exercises 1–2, suppose $f(x) = ab^x$.

1. Find a and b if $f(0) = \frac{8}{3}$ and $f(3) = \frac{1}{81}$.
2. Find $f(5)$ if $f(1) = 2$ and $f(2) = \frac{4}{3}$.
3. The value of a car depreciates by a fixed percent each year.
 - a. Write an equation that gives the current value V of the car in terms of the original value V_0 , the fixed percent p expressed as a decimal, and time t in years.
 - b. Suppose that the original value of the car was \$16,000, and after 2 years this value had depreciated to \$9000. Find the value of the car after 5 years, to the nearest dollar.
4. Carbon-14, an unstable isotope of carbon which decays exponentially to a more stable form, is used to date animal remains. After 5700 years, one-half of the original amount of carbon-14, by weight, remains.
 - a. Write an exponential function with base one-half relating the amount N of carbon-14 in the animal remains after t years to the original amount N_0 .
 - b. Suppose an animal contained 3.2 mg of carbon-14 when it was alive. Estimate how long ago the animal died, if its remains contain 0.4 mg of carbon-14 today.
5.
 - a. Describe the common features of all graphs of functions of the form $y = 3a^{-x}$ for all possible values of $0 < a < 1$.
 - b. Repeat part (a) for functions of the form $y = -5a^{-x}$.
6. Let $f(x) = b^x$. For a fixed positive integer n , let $g(x) = [f(x)]^n$, and let $h(x) = f(f(f(\dots(x)\dots)))$ (n compositions).
 - a. Express $g(x)$ as $f(?)$. (*Hint:* Express $g(x)$ in terms of b , n and x .)
 - b. Express $h(x)$ as $f(?)$.