LESSON 8.1

NAME _____ DATE _____

Challenge: Skills and Applications

For use with pages 465–472

In Exercises 1–4, write each function in the form $y = ab^x$.

1. $y = 4^{x+1}$

2. $y = 5 \cdot \sqrt{9^{x-1}}$

3. $y = \sqrt{5 \cdot 2^{3x}}$

4. $y = \dfrac{\sqrt{2}}{3 \cdot 5^{2x}}$

5. A *mortgage* is a loan that is paid off in equal monthly installments M, which include interest as well as repayment of the principal. The amount p_k of the kth installment that goes toward repayment of principal is given by the formula

$$p_k = \frac{M}{(1 + r)^{n-k+1}}$$

where the mortgage is fully repaid in n months and r is the monthly interest rate.

a. Suppose $r = 0.7\%$, $M = \$1050$, and $n = 300$. Find p_1 and p_{300}.

b. Is the formula a model of exponential growth?

6. a. Let $f(x) = 2^x$. Find the finite (first) differences between the values of f for $x = 0$, 1, 2, 3, 4, and 5. Write a formula that gives the difference between $f(x + 1)$ and $f(x)$ in terms of x.

b. Repeat part (a) but use the function $y = 3^x$.

c. Repeat part (a) for the function $y = 5^x$.

d. Make a conjecture for a general formula that gives the difference between $f(x + 1)$ and $f(x)$ for the function $y = a^x$, for $a > 1$.

7. The *average growth rate* of an exponential function $f(x)$ on an interval $a \le x \le b$ is

$$\frac{f(b) - f(a)}{b - a}.$$

The *instantaneous growth rate at a* is the number that this fraction approaches (if any) as $b \to a$. Let $f(x) = 3^x$, and let $b = a + h$. Write the instantaneous growth rate at a as the product of 3^a and a function of h (that does not involve a). Use a calculator to estimate the number this second factor approaches as $h \to 0$.