

Challenge: Skills and Applications

For use with pages 445–452

1. The z -score corresponding to a data point x is the number z defined by

$$z = \frac{x - \bar{x}}{\sigma},$$

where \bar{x} is the mean of the data and σ is the standard deviation of the data.

- Find the z -score of each data point in the data: 10, 35, 50, 60, 85.
 - What is the mean of the z -scores? What is their standard deviation? Do you think these values will be the same for any complete set of z -scores?
2. The *median-median line*, like the best-fitting line, is a linear function that approximates the relationship between two sets of data. You will find the equation of the median-median line for the following data: x : 3, 6, 8, 10, 12, 15, 19, 20, 22; y : 14, 16, 17, 22, 28, 28, 32, 37, 40.
- Divide each set of data into 3 groups: the smallest third, the middle third, and the largest third (the data are already in order from smallest to largest), and find the 6 medians $x_1, x_2, x_3, y_1, y_2, y_3$, one for each of these groups.
 - Find an equation of the line passing through (x_1, y_1) and (x_3, y_3) , $y = mx + b$. Use this to find the coordinates of the point P on this line for $x = x_2$.
 - Suppose the y -coordinate of P is y' . Let $w = \frac{y_2 + 2y'}{3}$. Find an equation of the line with slope m passing through $Q(x_2, w)$. This is the median-median line.
3. The number σ^2 (the standard deviation “without the radical”) is called the *variance* of the data. Use the data x_1, x_2, x_3 and the following method to show how σ^2 can be written in simplest form.
- Let \bar{x} be the mean of the data and let $\overline{x^2}$ represent the mean of the *squares* of the data (i.e. the mean of the data set that consists of the squares of the given set). Show that

$$\sigma^2 = \frac{1}{3}[x_1^2 + x_2^2 + x_3^2 - 2\bar{x} \cdot (x_1 + x_2 + x_3) + 3(\bar{x})^2].$$
 - Show that the right side of the equation in part (a) simplifies to $\overline{x^2} - (\bar{x})^2$.