LESSON **6.3**

NAME ______________________ DATE __________

# *Challenge: Skills and Applications*

**For use with pages 338–344**

**Simplify.**

**1.** $(p + q)(p - q)(p^2 + q^2)$

**2.** $(x - y)(x + y)(x^4 + x^2y^2 + y^4)$

**3.** $(a + b - c)(a + b + c)$

**4.** $(u^2 + v - w)(u^2 - v + w)$

**5. a.** Simplify the product $(x + y)(x^2 - xy + y^2)$.

**b.** Simplify the product $(x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$.

**c.** Based on your answers to parts (a) and (b), write a general formula. Use "$2n$" to represent a general even integer and "$2n + 1$" to represent a general odd integer, and use ". . ." for missing terms.

**6. a.** Expand $(a + b)^4$.

**b.** Use the formula you wrote in part (a) to simplify $(x^2 + 2)^4$.

**c.** Multiply the expanded form you found in part (a) by $(a + b)$ to get a formula for $(a + b)^5$. When combining like terms, express each coefficient (except the first and last) *as a sum of 2 coefficients* from the expansion of $(a + b)^4$. (For example, write "$(1 + 2)$" instead of "3").

**d.** Based on the way you wrote $(a + b)^5$ in part (c), state a rule for finding the coefficients of $(a + b)^n$, given the coefficients of $(a + b)^{n-1}$.

**7.** Explain how you know that $(n + 1)^k - n^k$ has degree $k - 1$.

**8.** In this problem, you will find a formula for $f(n) = 0^2 + 1^2 + 2^2 + 3^2 + \cdots + n^2$.

**a.** Assume that $f(n)$ can be expressed as a polynomial in the variable $n$ of degree 3: $f(n) = an^3 + bn^2 + cn + d$. Use $f(0)$ to explain why $d = 0$.

**b.** Explain why $f(n + 1) - f(n) = (n + 1)^2$.

**c.** Expand $f(n + 1) - f(n)$ as given in part (a).

**d.** Use the equation in part (b) to find $a$, $b$, and $c$ by equating the coefficients of like powers of $n$ on both sides. Write the formula you have discovered for $f(n)$.