LESSON 6.2

NAME ______________________________ DATE ____________

Challenge: Skills and Applications

For use with pages 329–336

Use synthetic substitution to evaluate each polynomial for the given complex value of *x*.

1. $x^3 - 5x^2 + 3x - 1;\ x = i$

2. $-x^3 + 2x^2 + 3x - 5;\ x = 1 + i$

3. $x^4 + x - 1;\ x = i$

4. $2x^4 + x^3 - 1;\ x = -i$

5. Suppose $f(x) = ax^n + bx^{n-1} + cx^{n-2} + \ldots$, and $g(x) = rx^n + sx^{n-1} + tx^{n-2} + \ldots$, where $r \neq 0$. In this problem you will prove that

$$\frac{f(x)}{g(x)} \text{ gets close to } \frac{a}{r} \text{ as } x \to \infty.$$

a. Write the fraction $\frac{f(x)}{g(x)}$ with each term of the numerator and denominator divided by x^n. How do you know that this new expression has the same values as the original fraction, provided $x \neq 0$?

b. Explain how you can easily tell what happens to the value of each term of the new expression as $x \to \infty$. Does the same argument work if you want to prove the same assertion with "$x \to \infty$" replaced by "$x \to -\infty$"?

6. You probably know that (*) a positive integer n is evenly divisible by 9 if and only if the sum of its digits is evenly divisible by 9. To prove this, write the following:

$$n = a \cdot 10^k + b \cdot 10^{k-1} + \ldots + r \cdot 10 + s$$

a. What do the numbers $a, b, \ldots, r, s$ represent? Explain how you know that for any positive integer j, $10^j = 9v + 1$ for some positive integer v.

b. Use the result of part (a) to show that you can "cast out 9s;" that is, the remainder when n is divided by 9 will be the same as the remainder when $a + b + \ldots + r + s$ is divided by 9.

c. Explain how your answer to part (b) proves the assertion (*).

7. a. Show that when the polynomial $P(x) = ax^3 + bx^2 + cx + d$ is evaluated for $x = k$ using synthetic substitution, the result is $ak^3 + bk^2 + ck + d$.

b. Use synthetic substitution and $P(x)$ in part (a) to show that if *a, b, c,* and *d* are *integers* and $P(2) = 0$, then *d* is an *even* integer.