

Challenge: Skills and Applications

For use with pages 170–175

1. An equation of a plane in space of the form $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is sometimes said to be in *point-normal form* (which is somewhat analogous to point-slope form for a line in a plane). The numbers a , b , and c are called *direction numbers* for the plane.
- Explain the “point” part of this name.
 - The line through the origin and the point (a, b, c) is perpendicular to this plane. (Such a line is said to be “normal” to the plane.) A set of parametric equations for this line is given by

$$x = at \quad y = bt \quad z = ct.$$

Find the value of t at which this line intersects the plane, in terms of a , b , c , x_1 , y_1 , and z_1 .

2. An equation of the form $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$ is said to be in *intercept form*.
- Explain this name.
 - Write the given equation in the form $ax + by + cz = k$, where a , b , c , and k are given in terms of p , q , and r .
 - Give a set of direction numbers a , b , and c (see Exercise 1), in terms of p , q , and r , such that the line through the origin and (a, b, c) is a line perpendicular to the plane with the above equation, as in part (b) of Exercise 1.
3. The numbers

$$y_1z_2 - y_2z_1, \quad x_2z_1 - x_1z_2, \quad x_1y_2 - x_2y_1$$

are direction numbers (see Exercise 1) for the plane that passes through the origin $(0, 0, 0)$, the point (x_1, y_1, z_1) , and the point (x_2, y_2, z_2) .

- Give an equation of the plane in point-normal form (see Exercise 1) in terms of x_1 , y_1 , z_1 , x_2 , y_2 , and z_2 , using the fact that the plane passes through the origin.
- Show that (x_1, y_1, z_1) is on the plane.