Extra Example 1 Finding the Slope of a Line

Find the slope of the line passing through $(-2, -4)$ and $(3, -1)$.

SOLUTION

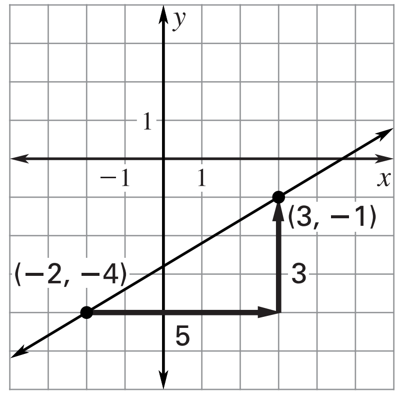
Let $(x_1, y_1) = (-2, -4)$ and $(x_2, y_2) = (3, -1)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ ← Rise: Difference in y-values ← Run: Difference in x-values

$= \dfrac{-1 - (-4)}{3 - (-2)}$ Substitute values.

$= \dfrac{-1 + 4}{3 + 2}$ Simplify.

$= \dfrac{3}{5}$ Simplify.



Extra Example 2 Classifying Lines by Slope

Without graphing, tell whether the line through the given points *rises*, *falls*, *is horizontal*, or *is vertical*.

a. $(-2, 3), (1, 5)$ **b.** $(1, -2), (3, -2)$

SOLUTION

a. $m = \dfrac{5 - 3}{1 - (-2)} = \dfrac{2}{3}$ Since $m > 0$, the line rises.

b. $m = \dfrac{-2 - (-2)}{3 - 1} = \dfrac{0}{2}$ Since $m = 0$, the line is horizontal.

Extra Example 3 Comparing Steepness of Lines

Tell which line is steeper.

Line 1: through $(1, -4)$ and $(5, 2)$ Line 2: through $(-2, -5)$ and $(1, -2)$

SOLUTION

The slope of Line 1 is $m_1 = \dfrac{2 - (-4)}{5 - 1} = \dfrac{2 + 4}{5 - 1} = \dfrac{6}{4} = \dfrac{3}{2}$.

The slope of Line 2 is $m_2 = \dfrac{-2 - (-5)}{1 - (-2)} = \dfrac{-2 + 5}{1 + 2} = \dfrac{3}{3} = 1$.

➧ Because the lines have positive slopes and $m_1 > m_2$, line 1 is steeper than line 2.

Extra Example 4 Classifying Parallel and Perpendicular Lines

Tell whether the two lines are *parallel*, *perpendicular*, or *neither*.

a. Line 1: through (1, −2) and (3, −2)
Line 2: through (−5, 4) and (0, 4)

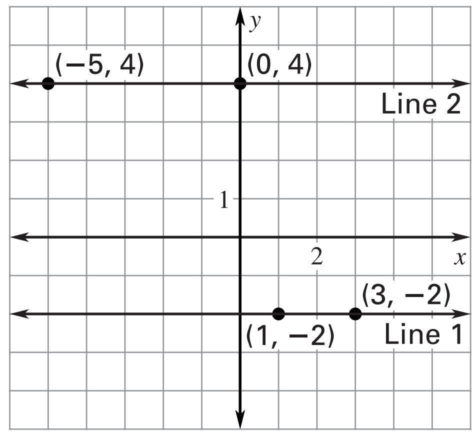
b. Line 1: through (−2, −2) and (4, 1)
Line 2: through (−3, −3) and (1, 5)

SOLUTION

a. The slopes of the two lines are:

$$m_1 = \frac{-2 - (-2)}{3 - 1} = \frac{0}{2} = 0$$

$$m_2 = \frac{4 - 4}{0 - (-5)} = \frac{0}{5} = 0$$

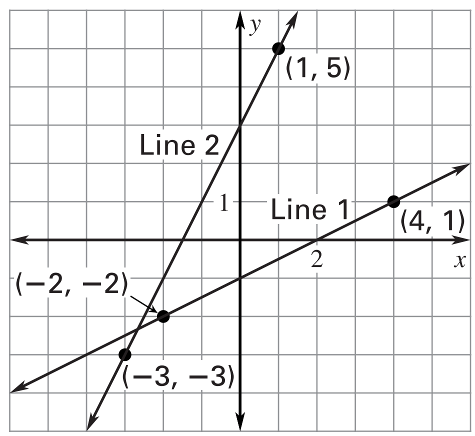


➧ Because $m_1 = m_2$, the slopes are the same. Since the lines are different, you can conclude that the lines are parallel.

b. The slopes of the two lines are:

$$m_1 = \frac{1 - (-2)}{4 - (-2)} = \frac{3}{6} = \frac{1}{2}$$

$$m_2 = \frac{5 - (-3)}{1 - (-3)} = \frac{8}{4} = 2$$



➧ The slopes are not the same, nor are they negative reciprocals of each other. Therefore you can conclude that the lines are neither parallel nor perpendicular.

Extra Example 5 Geometrical Use of Slope

The slope of a road, or grade, is usually expressed as a percent. For example, if a road has a grade of 3%, it rises 3 feet for every 100 feet of horizontal distance.

a. Find the grade of a road that rises 75 feet over a horizontal distance of 2000 feet.

b. Find the horizontal length x of a road with a grade of 4% if the road rises 50 feet over its length.

SOLUTION

a. The grade of a road that rises 75 feet over a horizontal distance of 2000 feet has slope $m = \frac{\text{rise}}{\text{run}} = \frac{75}{2000} = .0375$. Therefore, the grade is $.0375 \times 100 = 3.75\ \%$.

b. To find x, the horizontal length of the road, use the grade to write a proportion.

$\frac{\text{rise}}{\text{run}} = \frac{4}{100}$	Write a proportion.
$\frac{50}{x} = \frac{4}{100}$	The rise is 50 and the run is x.
$5000 = 4x$	Cross multiply.
$1250 = x$	Solve for x.

➧ So, the horizontal length of the road is 1250 ft.

Extra Example 6 Slope as a Rate of Change

CELLULAR PHONES The number of U.S. cell phone subscribers increased from 16 million in 1993 to 44 million in 1996. Find the average rate of change and use it to estimate the number of subscribers in 1997.

SOLUTION

$$\text{Average rate of change} = \frac{\text{Change in subscribers (in millions)}}{\text{Change in time}}$$

$$= \frac{44 - 16}{1996 - 1993} = \frac{28}{3} \approx 9.3$$

The average rate of change is an increase of 9.3 million subscribers per year. In 1997, there will be about $44 + 9.3 = 53.3$ million subscribers.