

Challenge: Skills and Applications

For use with pages 67–74

In Exercises 1–5, use the following information.A function f may have the property that, for *all* numbers a and b in the domain of f ,

$$f(a) + f(b) = f(a + b).$$

Check whether each of the following functions has the property described above. If it does not, give an example to show that it does not.

1. $f(x) = x^2$
2. $f(x) = 2x$
3. $f(x) = |x|$
4. $f(x) = 5x + 1$
5. What must be true about the numbers m and/or b in order for a function of the form $f(x) = mx + b$ to have the property above?

In Exercises 6–10, use the following information.A function with the property that $f(-x) = f(x)$, for all numbers x in its domain, is called an *even* function. A function with the property that $f(-x) = -f(x)$, for all x , is called an *odd* function. Tell whether each of the following is odd or even or neither.

6. $f(x) = x^2 + 2$
7. $x + 1$
8. $f(x) = f(x) = x^3 + x$
9. $f(x) = |x|$
10. If $f(x)$ is a function of the form $f(x) = ax^n + bx^{n-1} + \dots + nx + p$, what do you think must be true about the coefficients, a, b, \dots, p in order for f to be an even function? an odd function?

In Exercises 11–14, use the following information.A *one-to-one* function f with the property that a and b are any two distinct numbers in the domain of f , $f(a) \neq f(b)$. (In other words, f never maps two distinct inputs to the same output.) Tell whether each of the following functions is one-to-one. If it is not, give an example of a violation of the foregoing definition.

11. $f(x) = x^3$
12. $f(x) = x^2 + 1$
13. $f(x) = |x|$
14. $f(x) = -2x + 5$