

Challenge: Skills and Applications

For use with pages 604–609

In Exercises 1–6, use substitution to factor the expression.**Example:** $x + 3x^{1/2} - 28$

Solution: Since $x = (x^{1/2})^2$, you can rewrite the expression as $(x^{1/2})^2 + 3x^{1/2} - 28$. Then you can do the substitution $y = x^{1/2}$. The polynomial becomes $y^2 + 3y - 28 = (y - 4)(y + 7)$.
So, $x + 3x^{1/2} - 28 = (x^{1/2} - 4)(x^{1/2} + 7)$.

1. $(x^{1/2})^2 - 7x^{1/2} + 10$
2. $x^6 - 4x^3 - 12$
3. $x - 4\sqrt{x} - 45$
4. $x + 11\sqrt{x} + 10$
5. $\frac{1}{x^2} - \frac{15}{x} + 56$
6. $\frac{1}{x^2} + \frac{3}{x} - 108$

In Exercises 7–10, use substitution to factor and then solve the equation.

7. $x^4 - 13x^2 + 36 = 0$
8. $x^4 - 11x^2 + 24 = 0$
9. $\frac{1}{x^2} + \frac{1}{6x} - \frac{1}{6} = 0$
10. $\frac{1}{x^2} + \frac{13}{3x} + \frac{10}{3} = 0$
11. How many solutions does $x^4 - 13x^2 - 48$ have? What are they?

In Exercises 12–14, use the following information.

A square garden has a sidewalk around its outer edge. The sidewalk is a little wider on one set of opposite sides than on the other set. The area inside the sidewalk is given by the trinomial $x^2 - 13.6x + 46.2$, where x is the length and width of the entire garden, in feet.

12. Factor $x^2 - 13.6x + 46.2$ (*Hint:* Use the quadratic formula or a graph.)
13. What are the widths of the sidewalks?
14. If the area inside the sidewalks is 2830.2 square feet, what are the dimensions of the garden?

